Correction exercice

Exercice 1

$25 \in \mathbb{N}$; $-12 \in \mathbb{Z}$; $-5{,}2 \in \mathbb{D}$; $\frac{5}{4} \in \mathbb{D}$

$\frac{2}{3} \in \mathbb{Q}$; $-\frac{12}{3} \in \mathbb{Z}$; $(\sqrt{3}+2) \in \mathbb{R}$; $\frac{\pi}{3} \in \mathbb{R}$; $\frac{\sqrt{81}}{3} = 3 \in \mathbb{N}$

Exercice 2

1) $A = \frac{5}{2} + \frac{8}{3} = \frac{15}{6} + \frac{16}{6} = \underline{\frac{31}{6}}$

$B = 2 + \frac{5}{7} = \frac{14}{7} + \frac{5}{7} = \underline{\frac{19}{7}}$

$C = \frac{1}{2/5} = \underline{\frac{5}{2}}$

$D = \frac{4/2}{6} = \frac{2}{6} = \underline{\frac{1}{3}}$

$E = \frac{7/9}{14/27} = \frac{7}{9} \times \frac{9\times 3}{7\times 2} = \underline{\frac{3}{2}}$

$F = 1 + \frac{1}{3} \times \frac{5}{2-\frac{5}{3}} = 1 + \frac{1}{3} \times \frac{5}{1/3} = 1 + \frac{1}{3} \times 5 \times 3$

$\underline{F = 6}$

2) $J = \frac{1}{x} + \frac{3}{x+2}$ On cherche les valeur où le(s) dénominateur(s) s'annulent

Pour tout $x \neq 0$ et $x \neq -2$ on a $J = \frac{(x+2)+3x}{x(x+2)} = \underline{\underline{\frac{4x+2}{x(x+2)}}}$

Pour tout $x \neq \frac{2}{3}$ et $x \neq -\frac{2}{3}$ on a $K = \frac{1}{2-3x} - \frac{1}{2+3x} = \frac{2+3x-(2-3x)}{(2-3x)(2+3x)}$

$$\underline{\underline{K = \frac{6x}{4-9x^2}}}$$

Pour tout $x \neq -2$ on a $M = 5 + \frac{3}{2+x} = \frac{5(2+x)+3}{2+x} = \underline{\underline{\frac{13+5x}{2+x}}}$

Pour tout $x \neq -1$ on a $N = 2 + \frac{\frac{1}{3}x}{x+1} = \frac{2(x+1)+\frac{1}{3}x}{x+1} = \frac{\frac{7}{3}x+2}{x+1} = \underline{\underline{\frac{7x+6}{3(x+1)}}}$

Pour tout $x \neq 0$ on a $P = 1 - \frac{\frac{3}{2}(x+1)}{x} = 1 - \frac{3(x+1)}{2} = \frac{2-3x-1}{2} = \underline{\underline{-\frac{3x+1}{2}}}$

Pour tout $x \neq 0$ et $x \neq -1$ (car $x^2+x=0$
$x(x+1)=0$ donc $x=0$ ou $x=-1$)

On a $R = \frac{2(x+1)-(x-3)(x+1)}{x^2+x} = \frac{(x+1)(2-(x-3))}{x(x+1)} = \frac{-x-1}{x} = \underline{\underline{-\frac{x+1}{x}}}$

Pas $x-3=0$ et $x+1=0$ et $x^2-9=0$

$x=3$ $\quad$ $x=-1$ $\quad$ $(x-3)(x+3)=0$

$x=3 \quad x=-3$

Pour tout $x \neq -3$; $x \neq -1$ et $x \neq 3$ on a :

$$S = \frac{x^2+x}{x-3} \times \frac{x^2-9}{x+1} = \frac{x(x+1)(x-3)(x+3)}{(x-3)(x+1)} = \underline{\underline{x(x+3)}}$$

Exercice 3

$A = \sqrt{27} \times 5\sqrt{6} = \sqrt{9\times 3} \times 5\sqrt{6} = 3\sqrt{3} \times 5\sqrt{6} = 15\sqrt{3\times 6} = 15\sqrt{9\times 2} = \underline{\underline{45\sqrt{2}}}$

$B = 7\sqrt{75} - 2\sqrt{12} = 7\sqrt{25\times 3} - 2\sqrt{4\times 3} = 35\sqrt{3} - 4\sqrt{3} = \underline{\underline{31\sqrt{3}}}$

$C = 2\sqrt{5} + \sqrt{0{,}0045} = 2\sqrt{5} + \sqrt{45\times 10^{-4}} = 2\sqrt{5} + 10^{-2}\sqrt{9\times 5} = 2\sqrt{5} + 3\times 10^{-2}\sqrt{5} = \underline{\underline{2{,}03\sqrt{5}}}$

$D = (11\sqrt{5} - 5\sqrt{11})(11\sqrt{5} + 5\sqrt{11}) = (11\sqrt{5})^2 - (5\sqrt{11})^2 = 121\times 5 - 25\times 11$

Exercice 4

1) À la calculatrice $X = 2\sqrt{7}$

2) $X^2 = \left(\sqrt{10-\sqrt{84}} + \sqrt{10+\sqrt{84}}\right)^2 = \left(\sqrt{10-\sqrt{84}}\right)^2 + 2\times\sqrt{10-\sqrt{84}} \times \sqrt{10+\sqrt{84}} + \left(\sqrt{10+\sqrt{84}}\right)^2$

$X^2 = 10 \cancel{-\sqrt{84}} + 2\times\sqrt{(10-\sqrt{84})(10+\sqrt{84})} + 10 + \cancel{\sqrt{84}}$

$X^2 = 20 + 2\times\sqrt{10^2 - (\sqrt{84})^2}$

$X^2 = 20 + 2\sqrt{100-84}$

$X^2 = 20 + 2\times\sqrt{16}$

$X^2 = 20 + 2\times 4$

$X^2 = 28$ donc $X = \sqrt{28}$ ou $X = -\sqrt{28}$

donc $X = 2\sqrt{7}$ ou $X = -2\sqrt{7}$ mais X est la somme de deux nombres positifs donc X est positif donc $\underline{\underline{X = 2\sqrt{7}}}$

Exercice 5

$$a = \frac{7}{2\sqrt{3}} = \frac{7\sqrt{3}}{2\sqrt{3}\times\sqrt{3}} = \underline{\underline{\frac{7\sqrt{3}}{6}}}$$

$$b = \frac{14}{3\sqrt{7}} = \frac{14\times\sqrt{7}}{3\sqrt{7}\times\sqrt{7}} = \frac{7\times2\sqrt{7}}{3\times7} = \underline{\underline{\frac{2\sqrt{7}}{3}}}$$

$$C = \frac{1}{2+\sqrt{5}} = \frac{1(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} = \frac{2-\sqrt{5}}{2^2-\sqrt{5}^2} = \underline{\underline{\frac{2-\sqrt{5}}{-1}}}$$

$$D = \frac{2+\sqrt{10}}{1+\sqrt{10}} = \frac{(2+\sqrt{10})(1-\sqrt{10})}{(1+\sqrt{10})(1-\sqrt{10})} = \frac{2-2\sqrt{10}+\sqrt{10}-10}{1^2-\sqrt{10}^2} = \frac{-8-\sqrt{10}}{-9} = \underline{\underline{\frac{8+\sqrt{10}}{9}}}$$

$$E = \frac{2}{4-\sqrt{2}} = \frac{2(4+\sqrt{2})}{(4-\sqrt{2})(4+\sqrt{2})} = \frac{2(4+\sqrt{2})}{4^2-\sqrt{2}^2} = \frac{2(4+\sqrt{2})}{2\times7} = \underline{\underline{\frac{4+\sqrt{2}}{7}}}$$

$$F = \frac{1-\sqrt{2}}{\sqrt{2}-\sqrt{3}} = \frac{(1-\sqrt{2})(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} = \frac{\sqrt{2}+\sqrt{3}-2-\sqrt{6}}{\sqrt{2}^2-\sqrt{3}^2} = \underline{\underline{2-\sqrt{2}-\sqrt{3}+\sqrt{6}}}$$

$$g = \frac{\sqrt{3}x}{\sqrt{2}} - \frac{\sqrt{2}x}{\sqrt{3}} = \frac{\sqrt{3}x\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} - \frac{\sqrt{2}x\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{\sqrt{6}x}{2} - \frac{\sqrt{6}x}{3} = \frac{3\sqrt{6}x-2\sqrt{6}x}{6} = \underline{\underline{\frac{\sqrt{6}x}{6}}}$$

$$H = \frac{1}{\sqrt{2}-2} + \frac{3}{\sqrt{3}} = \frac{(\sqrt{2}+2)}{(\sqrt{2}-2)(\sqrt{2}+2)} + \frac{3\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{\sqrt{2}+2}{-2} + \frac{3\sqrt{3}}{3} = \frac{-3\sqrt{2}-6+6\sqrt{3}}{6}$$

$$H = \frac{3(-\sqrt{2}-2+2\sqrt{3})}{3\times2} = \underline{\underline{\frac{2\sqrt{3}-\sqrt{2}-2}{2}}}$$

$$I = \frac{1}{2-\sqrt{2}} - \frac{1}{2+\sqrt{2}} = \frac{1(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} - \frac{1(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})} = \frac{2+\sqrt{2}}{4-2} - \frac{2-\sqrt{2}}{4-2}$$

$$I = \frac{2\sqrt{2}}{2} = \underline{\underline{\sqrt{2}}}$$

Exercice 6

$$A = a^2\times a^5\times a^{-3} = a^{2+5-3} = \underline{\underline{a^4}}$$

$$B = (a^{-2})^3\times a = a^{-2\times3}\times a = a^{-6+1} = \underline{\underline{a^{-5}}}$$

$$C = (-2x^5)^{-4} = +\,2^{-4}\,x^{-5\times(-4)} = \underline{\underline{\frac{1}{16}x^{20}}}$$

$$D = \frac{a^5 b^{-4}}{a^{-5} b^{-2}} = a^{5-(-5)} b^{-4-(-2)} = \underline{\underline{a^{10} b^{-2}}}$$

$$E = \frac{16^{-4} \times 3^{21}}{6^3 \times 9^7} = \frac{(2^4)^{-4} \times 3^{21}}{(2\times 3)^3 \times (3^2)^7} = \frac{2^{4\times(-4)} \times 3^{21}}{2^3 \times 3^3 \times 3^{2\times 7}} = 2^{-16-3} \times 3^{21-3-14}$$

donc $E = \underline{\underline{2^{-19} \times 3^4}}$

$$F = (a^{-5} b^2)^{-1} \times ab^{-3} = a^{-5\times(-1)} b^{2\times(-1)} \times ab^{-3} = a^5 b^{-2} a b^{-3} = \underline{\underline{a^6 b^{-5}}}$$

$$G = \frac{2^{-5} \times (-6)^3 \times 3^{-4}}{-9^{-2} \times 8^{-4}} = \frac{2^{-5} \times (-6^3) \times 3^{-4}}{-(3^2)^{-2} \times (2^3)^{-4}}$$

$$= \frac{2^{-5} \times 2^3 \times 3^3 \times 3^{-4}}{3^{-4} \times 2^{-12}}$$

$$= 2^{-5+3-(-12)} \times 3^{3-4-(-4)}$$

$$\underline{\underline{G = 2^{10} \times 3^3}}$$