Correction DST 1

Exercice 1 :

$$A = \frac{15}{16} \times \frac{40}{3} \times \frac{18}{75}$$

$$A = \frac{\cancel{5}\times\cancel{3}\times\cancel{8}\times\cancel{5}\times\cancel{3}\times\cancel{3}\times\cancel{2}}{\cancel{2}\times\cancel{8}\times\cancel{3}\times\cancel{3}\times\cancel{5}\times\cancel{5}}$$

$$\underline{A = 3}$$

$$B = \frac{7}{2} + \frac{5}{2} \times \left(\frac{12}{25} - \frac{4}{5}\right)$$

$$B = \frac{7}{2} + \frac{5}{2} \times \left(-\frac{8}{25}\right)$$

$$B = \frac{7}{2} - \frac{\cancel{8}\times 2\times 4}{\cancel{2}\times\cancel{5}\times 5}$$

$$B = \frac{35}{10} - \frac{8}{10}$$

$$\underline{B = \frac{27}{10}}$$

$$C = \left(\frac{7}{12} - 1\right)\left(\frac{7}{12} + 1\right)$$

$$C = \frac{49}{144} - 1$$

$$\underline{C = -\frac{95}{144}}$$

$$D = \left(-\frac{4}{5}\right)^2 + \frac{4}{5}$$

$$D = \frac{16}{25} + \frac{20}{25}$$

$$\underline{D = \frac{36}{25}}$$

$$E = \frac{\frac{1}{2} + \frac{4}{5}}{\frac{1}{2} - \frac{6}{7}}$$

$$E = \frac{\frac{13}{10}}{-\frac{5}{14}}$$

$$E = \frac{13}{10} \times \left(-\frac{14}{5}\right)$$

$$E = -\frac{13\times 2\times 7}{5\times 2\times 5}$$

$$\underline{E = -\frac{91}{25}}$$

$$F = -3^2 + \frac{\frac{14}{5}}{\frac{28}{15}}$$

$$F = -9 + \frac{\cancel{7}\times\cancel{2}\times 3\times\cancel{5}}{\cancel{5}\times\cancel{7}\times\cancel{2}\times\cancel{2}}$$

$$F = -9 + \frac{3}{2}$$

$$\underline{F = -\frac{15}{2}}$$

2) $G = -(-2)^2 \times (-24)^5$

$= -2^2 \times (-3 \times 2^3)^5$

$= -2^2 \times (-3^5 \times 2^{15})$

$= 2^2 \times 3^5 \times 2^{15}$

$\underline{G = 2^{17} \times 3^5}$

$H = \dfrac{(5 \times 11)^3 \times 5^{-7} \times 11^4}{(-11)^6 \times 55^{10}}$

$H = \dfrac{5^3 \times 11^6 \times 5^{-7} \times 11^4}{11^6 \times 5^{10} \times 11^{10}}$

$H = 5^{3-7-10} \times 11^{6+4-6-10}$

$\underline{H = 5^{-14} \times 11^{-6}}$

$K = \left(\frac{7}{3}\right)^{-5} \times \frac{27}{49} \times \frac{(7^{10})^{-1}}{9^5}$

$K = \dfrac{7^{-5} \times 3^3 \times 7^{-10}}{3^{-5} \times 7^2 \times (3^2)^5}$

$K = \dfrac{7^{-15} \times 3^3}{3^{-5} \times 7^2 \times 3^{10}}$

$K = 7^{-15-2} \times 3^{3+5-10}$

$\underline{K = 7^{-17} \times 3^{-2}}$

$L = \dfrac{a^{-2} \times b^8}{ab^3} \times \dfrac{(a^{-1}b^2)^4}{aba^{-1}}$

$L = \dfrac{a^{-2} \times b^8 \times a^{-4} \times b^8}{ab^3 \times aba^{-1}}$

$L = a^{-2-4-1-1+1} \times b^{8+8-3-1}$

$\underline{L = a^{-7} \times b^{12}}$

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3) $M = (5 + \sqrt{3})^2$

$M = 25 + 10\sqrt{3} + 3$

$\underline{M = 28 + 10\sqrt{3}}$

$N = (2 - \sqrt{5})(\sqrt{5} + 4)$

$N = 2\sqrt{5} + 8 - 5 - 4\sqrt{5}$

$\underline{N = 3 - 2\sqrt{5}}$

$P = 5\sqrt{10} - 11\sqrt{10} + 2\sqrt{490} + (\sqrt{2} - 3\sqrt{5})$

$P = -6\sqrt{10} + 2 \times \sqrt{49} \times \sqrt{10} + 2 - 6\sqrt{10} + 45$

$P = -6\sqrt{10} + 14\sqrt{10} - 6\sqrt{10} + 47$

$\underline{P = 2\sqrt{10} + 47}$

$Q = \dfrac{-8}{\sqrt{7}} + 3\sqrt{28}$

$Q = -\dfrac{8\sqrt{7}}{7} + 3 \times 2\sqrt{7}$

$Q = -\dfrac{8\sqrt{7}}{7} + 6\sqrt{7}$

$\underline{Q = \dfrac{34}{7}\sqrt{7}}$

$$R = \frac{5}{\sqrt{3}-1}$$

$$R = \frac{5\sqrt{3}+5}{3-1}$$

$$R = \frac{5\sqrt{3}+5}{2}$$

$$R = \frac{5}{2}\sqrt{3} + \frac{5}{2}$$

4) $A = 3$ donc $A \in \mathbb{N}$

$D = \frac{36}{25}$ donc $D = \frac{144}{10^2}$ donc $D \in \mathbb{D}$

$F = -\frac{15}{2}$ donc $F = -\frac{75}{10}$ donc $F \in \mathbb{D}$

$K = 7^{-17} \times 3^{-2}$ donc $K = \frac{1}{7^{17} \times 3^2}$ donc $F \in \mathbb{Q}$

$R = \frac{5}{2}\sqrt{3} + \frac{5}{2}$ donc $R \in \mathbb{R}$

Exercice 2

1) $A = (3x+2)(-x+5) - (x+7)^2$

$A = -3x^2 + 15x - 2x + 10 - (x^2 + 14x + 49)$

$A = -3x^2 + 15x - 2x + 10 - x^2 - 14x - 49$

$A = -4x^2 - x - 39$

$B = (12x-5)(12x+5)$

$B = 144x^2 - 25$

$C = (-4x+1)(5x+7) + 3(9-x)(x+10)$

$C = -20x^2 - 28x + 5x + 7 + 3(9x + 90 - x^2 - 10x)$

$C = -20x^2 - 23x + 7 + 27x + 270 - 3x^2 - 30x$

$\underline{C = -23x^2 - 26x + 277}$

$D = (3x+1)^3$

$D = (3x+1)^2(3x+1)$

$D = (9x^2 + 6x + 1)(3x+1)$

$D = 27x^3 + 9x^2 + 18x^2 + 6x + 3x + 1$

$\underline{D = 27x^3 + 27x^2 + 9x + 1}$

2) $E = (-x+5)(2x-3) + (2x-3)(4x+11)$

$E = (2x-3)(-x+5+4x+11)$

$\underline{E = (2x-3)(3x+16)}$

$F = 25x^2 - 81$

$F = (5x)^2 - 9^2$

$\underline{F = (5x-9)(5x+9)}$

$G = 8x^3 - 8x^2 + 2x$

$G = 2x(4x^2 - 4x + 1)$

$\underline{G = 2x(2x-1)^2}$

$H = 100x^2 - 121 + (10x+11)(-3x-7)$

$H = (10x)^2 - 11^2 + (10x+11)(-3x-7)$

$H = (10x+11)(10x-11) + (10x+11)(-3x-7)$

$H = (10x+11)(10x-11 + (-3x-7))$

$\underline{H = (10x+11)(7x-18)}$

$K = (-2x+11)^2 - (7x-3)^2$

$K = (-2x+11+7x-3)(-2x+11-7x+3)$

$\underline{K = (5x+8)(-9x+14)}$

$L = (3x+8)^2 - 9(x-5)^2$

$L = (3x+8)^2 - (3x-15)^2$

$L = (3x+8-3x+15)(3x+8+3x-15)$

$\underline{L = 23(6x-7)}$

$M = (2x+6)(1-x) - (x+1)(5x+15)$

$M = 2(x+3)(1-x) - (x+1)\times 5(x+3)$

$M = (x+3)(2-2x-5x-5)$

$M = (x+3)(-3-7x)$

$\underline{M = -(x+3)(7x+3)}$

3) Pour $x \neq -1$ et $x \neq -2$, $\frac{3x}{x+1} - \frac{x}{x+2} = \frac{3x(x+2) - x(x+1)}{(x+1)(x+2)}$

$= \frac{3x^2+6x-x^2-x}{(x+1)(x+2)}$

$\underline{N = \frac{2x^2+5x}{(x+1)(x+2)}}$

Pour $x \neq 0$ et $x \neq 3$, $P = \frac{7}{x(x-3)} + \frac{1}{x-3}$

$\underline{P = \frac{7+x}{x(x-3)}}$

Exercice 3

1) Dans le triangle NDB, le plus long côté est [BN]

$BN^2 = 13^2$ et $BD^2 + DN^2 = 12^2 + 5^2$

$BN^2 = 169$ $= 144 + 25$

$BD^2 + DN^2 = 169$

Donc $BN^2 = BD^2 + DN^2$

D'après la réciproque du théorème de Pythagore, le triangle NDB est rectangle en D.

2) Dans le triangle ABC rectangle en B, d'après le théorème de Pythagore, on a :

$AC^2 = AB^2 + BC^2$

$11^2 = 7^2 + BC^2$

$BC^2 = 121 - 49$

$BC^2 = 72$ et $BC \geq 0$

$BC = \sqrt{72}$

$\underline{BC = 6\sqrt{2}}$

3) Dans le triangle DEF rectangle en D, d'après le théorème de Pythagore, on a :

$EF^2 = ED^2 + DF^2$

$EF^2 = (3x)^2 + (x+1)^2$ et $x \geq 0$

$EF^2 = 9x^2 + x^2 + 2x + 1$

$EF^2 = 10x^2 + 2x + 1$ et $EF \geq 0$

$\underline{EF = \sqrt{10x^2 + 2x + 1}}$